Effect of Interlayer Thickness on Stress Singularity Field Near a Vertex in Three-Dimensional Joints Under Thermal and Mechanical Loadings

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Abstract. Influence of interlayer thickness on stress fields at the vertices in three-dimensional joints with three layers is evaluated using eigen analysis formulated by finite element method (FEM) and boundary element method (BEM). A model for analysis is a three-layered joint consisting of Si, resin and FR-4.5. Singular stress fields for mechanical loading and residual thermal stress are investigated. All stress components are expressed in spherical coordinate systems where their origins are located at the vertices of each interface. Expressions for stress field near the vertex on the interface are derived from the result of eigen analysis. The order of singularity at Si-resin interface is larger than that at resin-FR-4.5 interface. A coefficient of a power law term of stress distribution in a radial direction from the vertex increases with increasing the thickness of interlayer in cases of mechanical loading and residual thermal stress. A coefficient of a power law term in the angular function of stress obtained from BEM is almost agreed with the value obtained from the angular function derived by eigen analysis. Three-dimensional intensity of singularity at the vertex of interface is defined using the intensities of singularity in the radial direction and that in the angular direction. The three-dimensional intensity of singularity at the vertex varies with the interlayer thickness in a similar way to the intensity of singularity in the radial direction from the vertex.

1. Introduction

A lot of studies on stress singularity occurring at a vertex on an interface in two- and three-dimensional bonded joints have been carried out until now⁽¹⁾⁻⁽⁵⁾. A singular stress field at a vertex of interface in a three-dimensional joint has a three dimensional characteristic including the influence of stress singularity line, which is a cross line of interface and free-surface as shown in figure 1. Three-dimensional singular stress field can be expressed as $\sigma_{ij}(r, \theta, \phi) = K_{ij} r^{-\lambda} f_{ij}(\theta, \phi)$, where r represents the distance from an origin of singular stress field, λ the order of stress singularity, K_{ij} the intensity of singularity of stress components σ_{ij} , $f_{ij}(\theta, \phi)$ the angular function of stress components σ_{ij} ⁽⁴⁾. The order of stress singularity, λ , which characterizes the singular stress field is determined from eigen analysis.

The authors have been investigated on the evaluation of reliability of CSP (Chip Size Packaging), which is used in advanced electronic devices. CSP is a joint consisting of silicon, resin and composite materials. Delamination occurs frequently at the vertex of interface between silicon (Si) and resin.

The singular stress field of the residual thermal stress in three-dimensional joints, which Si and substrate (FR-4.5) are bonded with encapsulation resin (resin), was already reported⁽⁵⁾. In the present paper, a stress analysis in three-layered joints subjected to a mechanical loading is conducted. In

particular, the influence of resin thickness on the singular stress field is compared with the results in thermal residual stress analysis.

Finally, a relationship between the three-dimensional intensities of singularity at the vertex on each interface and the resin thickness is investigated, and the difference between the intensity of singularity for thermal residual stress and that for mechanical loading will be clarified.

Figure 1. Three-dimensional joint with three layers.

2. ANALYSIS METHOD FOR STRESS SINGULARITY FIELD

2.1. Boundary element method

Boundary integral equation can be expressed as follows.

$$c_{ij}(P)u_i(P) = \int_{\Gamma} \{U_{ij}(P,Q)t_j(Q) - T_{ij}(P,Q)u_j(Q)\} dS(Q) \quad (1)$$

where $c_{ij}(P)$ represents a constant depending on the shape of boundary, P and Q are an observation point and a source point locating on the boundary Γ , U_{ij} and T_{ij} represent the fundamental solutions for displacement and traction. Stress distribution in the domain for analysis is calculated using Hooke's law and the following equation.

$$u_{i,k}(P) = \int_{\Gamma} \{U_{ij,k}(P,Q)t_j(Q) - T_{ij,k}(P,Q)u_j(Q)\} dS(Q) \quad (2)$$

where $U_{ij,k}$ and $T_{ij,k}$ are derivatives of the fundamental solutions of displacement and traction with respect to P .

In this analysis, a mesh division on an interface is not needed since Rongved's solution⁽⁶⁾, which is a fundamental solution for two-phase materials, is used. Hence, the stress distribution near a singular stress point on the interface can be calculated accurately. The present analysis model is a three-layered bonded joint, so a domain method is employed in the BEM analysis.

2.2. Eigen analysis

The order of stress singularity, λ , which characterizes the singular stress field is determined using an eigen equation, which is formulated by a finite element method, as follows⁽⁷⁾.

$$(p^2[\mathbf{A}] + p[\mathbf{B}] + [\mathbf{C}])\{\mathbf{u}\} = 0 \quad (3)$$

where $\lambda = \text{Re}(p) - 1$, p is an eigen value in equation (3), $[\mathbf{A}]$, $[\mathbf{B}]$ and $[\mathbf{C}]$ are matrices consisting of elastic moduli and the geometry of joints, $\{\mathbf{u}\}$ represents a nodal displacement vector. In case of $-1 < \lambda < 0$, a stress singularity exists in the stress field, and in case of $\lambda > 0$, the stress singularity disappears.

3. ANALYSIS MODEL BOUNDARY CONDITIONS

In the present study, three-dimensional blocks of Si and FR4.5 are jointed by resin. The model for analysis is shown in figure 2. Considering the symmetry of the joint, a quarter model is analyzed using BEM. In order to clarify the influence of resin thickness on the stress singular field at the vertexes of two interfaces (Si and resin, resin and FR4.5), the resin thickness, t , is varied from 0.002 to 100mm ($t=0.002, 0.004, 0.01, 0.025, 0.1, 0.2, 0.4, 1, 2, 4, 10, 20, 40, 100\text{mm}$). Eight nodes quadrilateral serendipity element is used, and the minimum size in an element near the vertex is $0.01\mu\text{m}\times 0.01\mu\text{m}$. Material properties are shown in table 1.

Singular stress fields for a mechanical loading and residual thermal stress, respectively, are analyzed. A unit mechanical loading, $\sigma_0=1\text{MPa}$, is applied to the upper surface of Si, and the lower surface of FR-4.5 is fixed in the normal direction of the surface. Stresses in spherical coordinate systems with origins at the vertexes of each interface are investigated. In this analysis, stress components for mode I, II and III at a crack tip, $\sigma_{\theta\theta}$, $\sigma_{r\theta}$ and $\sigma_{\phi\theta}$ are investigated.

In the residual thermal stress analysis, an external force corresponding to thermal stress for each material property is applied to the side of the model and the stress analysis is conducted. After the analysis, the applied external force is removed from the result of stress analysis, and then the residual thermal stress is obtained. In the present analysis, ΔT is taken as -155K and material property does not vary with temperature. The values of the external force are shown in table 1.

Figure 2. Model for BEM analysis.

Table 1. Material properties used in analysis.

	Young's modulus	Thermal expansion	Poisson's ratio	External force
	E[GPa]	$\alpha [10^{-6}/\text{K}]$	ν	$\sigma_{xx}, \sigma_{yy} [\text{MPa}]$
Silicon	166.0	3.0	0.26	161.0
Resin	2.74	33.0	0.38	58.4
FR-4.5	15.34	14.0	0.15	47.6

4. Results of analysis

4.1. Eigen analysis

4.1.1. Order of stress singularity. A model for eigen analysis is shown in figure 3, where wedge angles at the vertex O are $\phi_1=\phi_2=90^\circ$, those at a point O_{line} on the stress singularity line are $\phi_1=\phi_2=180^\circ$. A mesh division on the developed $\theta \times \phi$ plane used in the eigen analysis is shown in figure 4. Mesh size is $\theta \times \phi = 9^\circ \times 9^\circ$, and furthermore, an element including the interface and the stress singularity line is equally divided by five, so $\theta \times \phi = 1.8^\circ \times 1.8^\circ$.

The order of stress singularity at the vertex O , λ_{vertex} , and that on the singularity line, λ_{line} , for each interface are shown in table 2. The order of stress singularity for Si-resin interface is larger than that for resin-FR-4.5 interface, and the value of λ_{vertex} is larger than that of λ_{line} for each interface. It is found that a triple root of eigen value $p=1$ at the vertex and a quadruple root of $p=1$ on the stress singularity line exist. In the previous study of three-dimensional joints, if triple and quintuple roots of $p=1$ exist, a logarithmic singularity may occur at the stress field. Then, the stress field at the vertex and on the stress singularity line can be expressed as follows.

$$\sigma_{ij}(\bar{r}, \theta, \phi) = \bar{K}_{1ij} f_{1ij}(\theta, \phi) \bar{r}^{-\lambda_{vertex}} + \bar{K}_{2ij} f_{2ij}(\theta, \phi) + \sum_{k=3}^M \bar{K}_{kij} f_{kij}(\theta, \phi) (\ln \bar{r})^{k-2} \quad (4)$$

where $M=4$ (with a triple root of $p=1$), $M=6$ (with a quintuple root of $p=1$). $\bar{K}_{1ij}$ represents the intensity of singularity in the r -direction and $\bar{r} = r/t$. Here, the angular function $f_{ij}(\theta, \phi)$ can be obtained from eigen vector ($\{\mathbf{u}\}$ in equation (3)) corresponding to eigen value p .

4.1.2. Angular functions of stress derived from eigen displacement vector. Spherical coordinate system with an origin located at a vertex on interface is shown in figure 5. Where r represents a distance from the origin O to an internal point P , r_0 represents the radius of sphere. Angular function for the spherical coordinate system can be obtained from eigen vector corresponding to eigen value, which is shown in table 2. The variation of angular function, $f_{1\theta\theta}$, with angle ϕ on each interface is shown in figure 6. The value of angular function is normalized by the value of $f_{1\theta\theta}(\pi/2, \pi/2)$. It is found that the value at Si-resin interface is larger than that at resin-FR-4.5 interface and a stress singular field exists at $\phi=0^\circ$ and 90° . Therefore, it is supposed that the angular function is affected by two stress singularity lines existing at side surfaces. Here, the angular function of stress on the interface can be expressed by the following equations.

Figure 3. A model for eigen analysis.

Figure 4. A mesh on the developed $\theta \times \phi$ plane.

Table 2. List of eigen value and order of stress singularity for vertex and line in joints.

	Silicon-resin interface				Resin-FR4.5 interface			
	Stress singularity point		Stress singularity line		Stress singularity point		Stress singularity line	
	p_{vertex}	λ_{vertex}	p_{line}	λ_{line}	p_{vertex}	λ_{line}	p_{line}	λ_{line}
1	1.000	0.000	1.000	0.000	1.000	0.000	1.000	0.000
2	1.000	0.000	1.000	0.000	1.000	0.000	1.000	0.000
3	1.000	0.000	1.000	0.000	1.000	0.000	1.000	0.000
4	0.605	0.395	1.000	0.000	0.781	0.219	1.000	0.000
5			1.000	0.000			1.000	0.000
6			0.682	0.318			0.821	0.179

$$f_{1\theta\theta}\left(\frac{\pi}{2}, \rho_A, \rho_B, \phi\right) = L_{1\theta\theta}^A \rho_A^{-\lambda_{line}} + L_{1\theta\theta}^B \rho_B^{-\lambda_{line}} + L_{2\theta\theta} \quad (5)$$

where $\rho_A = \rho \sin \phi$, $\rho_B = \rho \cos \phi$, and $\rho = r/r_0$. ρ_A is distance from the singularity line A and ρ_B is distance from the singularity line B.

Now, $L_{k\theta\theta}^A = L_{k\theta\theta}^B$, since the value of the order of stress singularity for the singularity line, OA, in the joint is the same as that for the singularity line, OB. Therefore, the angular function at the vertex on the interface can be written as follows.

$$f_{1\theta\theta}\left(\frac{\pi}{2}, \rho_A, \rho_B, \phi\right) = L_{1\theta\theta} (\rho_A^{-\lambda_{line}} + \rho_B^{-\lambda_{line}}) + L_{2\theta\theta} \quad (6)$$

where the coefficients, L_{1ij} , represent the intensity of singularity in the ϕ -direction, λ_{line} represents the order of stress singularity for the stress singularity line. Here, logarithmic terms are neglected. The coefficients, L_{1ij} , determined by approximating the distributions of angular functions by equation (5) are shown in table 3. $L_{k\theta\theta}^{Eigen}$ indicates the value of the eigen analysis. It is found that $L_{k\theta\theta}^{Eigen}$ for resin-FR4.5 interface is larger than that for Si-resin interface.

Figure 5. Spherical coordinate system with an origin at the singularity point.

Table 3. List of coefficients in angular function.

	Si-resin	Resin-FR-4.5
$L_{1\theta\theta}^{Eigen}$	0.687	0.763
$L_{2\theta\theta}^{Eigen}$	-0.570	-0.622

Figure 6. Distribution of angular function, $f_{1\theta\theta}$, at the vertex on each interface.

4.2. BEM results and the intensity of stress singularity

4.2.1. Distribution of stress in the r -direction at the vertex for each interface. The intensity of stress singularity in a radial direction of the vertex is determined from the stress distribution obtained using BEM. Distributions of stress, $\sigma_{\theta\theta}$, on each interface against a dimensionless distance r/t at $\phi=45^\circ$ are shown in figures 7(a) and (b) for various resin thicknesses. It found that the value of stress decreases with increasing the resin thickness, t . Comparing with the results of residual thermal stress⁽⁵⁾ the minimum values of stress take at $r/t=0.6$ for Si-resin interface, at $r/t=0.15$ for resin-FR-4.5 interface. Therefore, the range of singular stress field is restricted by the dimension of joint such as resin thickness. Singular stress field in the three-dimensional joints is represented by equation (4). For simplicity, the singular stress field is approximated by using the following equation including $t^{\lambda_{vertex}}$ in $\bar{K}_{1ij}$.

$$\sigma_{\theta\theta}(r, \theta, \phi) = K_{1\theta\theta} f_{1\theta\theta}(\theta, \phi) r^{-\lambda_{vertex}} + K_{2\theta\theta} f_{2\theta\theta}(\theta, \phi) + \sum_{k=3}^M K_{k\theta\theta} f_{k\theta\theta}(\theta, \phi) (\ln r)^{k-2} \quad (7)$$

where $\bar{K}_{1ij} t^{\lambda_{vertex}} = K_{1ij}$. Intensity of stress singularity, K_{1ij} , in equation (7) is determined from the curves shown in figure 7 using a least square method. Here, the angular function $f_{1\theta\theta}$ is normalized by the value at $\phi=45^\circ$ on the interface. The intensity of stress singularity, $K_{1\theta\theta}$, against t/W (t : the resin thickness, W : the width of the model) is shown in figure 8. The intensity of stress singularity can be expressed by two straight lines that have a bend point at $t/W=0.05$ ($t=1$ mm). In the results for residual thermal stress and external force, $K_{1\theta\theta}$ for resin-FR4.5 interface is larger than that for Si-resin interface. Stress distribution on Si-resin is similar to that on resin-FR-4.5 interface over $t/W=0.01$ ($t=0.2$ mm).

The value of $K_{1\theta\theta}$ for each interface takes a constant over $t/W > 1$. Therefore, the singular stress field is not affected in the resin thickness over the width of the model.

Figure 7. Distribution of stress, $\sigma_{\theta\theta}^{Mech}$, for external loading against r/t on each interface.

Figure 8. A relationship between $K_{1\theta\theta}$ and t/W of each condition.

4.2.2. Distribution of stress for ϕ -direction at the vertex of each interface. In this section, the stress distributions in the singular stress field on each interface are investigated against the angle ϕ . The stress distribution against ϕ is related to the angular function of stress obtained from eigen value analysis. However, the singular stress field in BEM is complicatedly affected by the resin thickness and singularity line. Distributions of stress, $\sigma_{\theta\theta}$, against ϕ at $r=0.0001$ mm on each interface are shown in figures 9(a) and (b). Stresses on each interface increase with increasing the resin thickness and stresses reach an upper limit at the resin thickness of $t=20$ mm.

The distribution of stress, $\sigma_{\theta\theta}$, for the external loading against ϕ is shown in figure 10. All plots are normalized by the value at $\phi=45^\circ$. It found that this distribution is very similar to the angular function obtained from eigen analysis. The normalized stress, $\sigma_{\theta\theta}$, is referred to as $g_{1\theta\theta}$, which is expressed by

$$g_{1\theta\theta}(\pi/2, \rho_A, \rho_B, \phi) = L_{1\theta\theta}^{BEM} \left(\rho_A^{-\lambda_{line}^A} + \rho_B^{-\lambda_{line}^B} \right) + L_{2\theta\theta}^{BEM} \quad (8)$$

Intensity of singularity, $L_{1\theta\theta}^{BEM}$, in equation (8) is determined from the plots shown in figure 10 using the least square method. The determined coefficient, $L_{1\theta\theta}^{BEM}$, is shown in figure 11. In the case of mechanical loading, the value of $L_{1\theta\theta}^{BEM}$ varies slightly with the resin thickness.

Figure 9. Distribution of stress, $\sigma_{\theta\theta}^{Mech}$, for mechanical loading against ϕ on each interface.

Figure 10. Distribution of normalized stress, $g_{1\theta\theta} = \sigma_{\theta\theta}^{Mech} / \sigma_{\theta\theta}^{Mech}|_{\phi=45^\circ}$, against ϕ on each interface.

Figure 11. A relationship between $L_{1\theta\theta}^{BEM}$ and t/W for each condition.

4.2.3. Three-dimensional intensity of stress singularity. The singular stress field on interface is expressed by equation (7), here $g_{1\theta\theta}$ is used as an angular function instead of $f_{1\theta\theta}$. Equation (7) can be modified as follows .

$$\sigma_{\theta\theta}(r, \theta, \phi) = K_{1\theta\theta} g_{1\theta\theta}(\theta, \phi) r^{-\lambda_{vertex}} + K_{2\theta\theta} g_{2\theta\theta}(\theta, \phi) \quad (9)$$

This equation represents the three-dimensional singular stress field associated with the intensities of singularity in the r - and the ϕ -directions.

$$\sigma_{\theta\theta}(r, \phi) = K_{1\theta\theta} L_{1\theta\theta}^{BEM} \left(\rho_A^{-\lambda_{ine}^A} + \rho_A^{-\lambda_{ine}^B} + \frac{L_{2\theta\theta}^{BEM}}{L_{1\theta\theta}^{BEM}} \right) r^{-\lambda_{vertex}} + K_{2\theta\theta} g_{2\theta\theta}(\theta, \phi) \quad (10)$$

Three-dimensional intensity of singularity is defined by a coefficient of power law term in equation (10) as $K_{1\theta\theta}^{3D} = K_{1\theta\theta} L_{1\theta\theta}^{BEM}$. Figure 12 represents a relationship between the three-dimensional intensity of singularity, $K_{1\theta\theta}^{3D}$, and the resin thickness, t . This figure demonstrates the existence of an upper limit of $K_{1\theta\theta}^{3D}$, which approaches to a constant value as the resin thickness attains to the width of the joint as well as $K_{1\theta\theta}$. Plots shown in the figure are approximated separately at $t/W=0.05$ by

$$K_{1\theta\theta}^{3D} = c(t/W)^n \quad (11)$$

where the values of c and n are shown in table 4. It found that the values in $0.05 < t/W \leq 1.0$ are larger than those in $t/W \leq 0.05$. It was found that a simple relationship between the three-dimensional intensities of singularity for the mechanical loading and residual thermal stress and the interlayer thickness exists such as equation (11).

Figure 12. A relationship between three-dimensional intensity of singularity $K_{1\theta\theta}^{3D}$ and t/W of each condition.

Table 4. Coefficients in equation (11).

		Residual thermal stress		Mechanical loading	
		$t/W \leq 0.05$	$0.05 < t/W \leq 1.0$	$t/W \leq 0.05$	$0.05 < t/W \leq 1.0$
Si-resin	c	0.00217	0.00368	0.0581	0.0921
	n	0.491	0.703	0.233	0.391
Resin-FR4.5	c	0.00391	0.0114	0.0318	0.0848
	n	0.177	0.554	0.0863	0.412

5. Conclusion

Singular stress analysis in the three-layered joints was conducted for various values of resin thickness under an external tensile load. A relationship between three-dimensional intensity of singularity and resin thickness was investigated. Finally, the intensity of singularity for residual thermal stress was compared with that for external tensile load. Results in this study are summarized as follows.

- (1) The order of stress singularity at the vertex on Si-resin interface was larger than that on resin-FR4.5 interface.
- (2) The coefficient of power law term, $K_{1\theta\theta}$, in the expression for the stress distribution in a radial direction obtained by BEM increases as the resin thickness increases. The value of $K_{1\theta\theta}$ reached an upper limit at the resin thickness of $t=20\text{mm}$, which was equal to the width of the joint.
- (3) The coefficient of power law term, $L_{1\theta\theta}^{BEM}$, in the normalized stress, $g_{1\theta\theta}$, on interface against the angle ϕ was investigated. The value of $L_{1\theta\theta}^{BEM}$ for resin-FR-4.5 interface was larger than that for Si-resin interface.
- (4) Intensity of singularity at the vertex of three-dimensional joint, $K_{1\theta\theta}^{3D}$, varied with resin thickness in the same manner as $K_{1\theta\theta}$. The upper limit of $K_{1\theta\theta}^{3D}$ exists at $t/W=1$. Furthermore, it was shown that there was a simple relationship between $K_{1\theta\theta}^{3D}$ and the resin thickness.

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